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Weak preopen sets and weak bicontinuity in texture spaces

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Abstract. The aim of this paper is introduce and study the notion of weak preopen sets and weak prebicontinuity on weak structures in texture spaces. It is presented some characterizations of weak prebicontinuity, and a link is given between weak spaces and weak structure on discrete texture spaces.

1. Introduction

In 2015, Min and Kim [16] have defined weak structures and w-spaces. Recall that a subfamily w_X of the power set $\mathcal{P}(X)$ of a nonempty set X is called *weak structure* on X, if $\emptyset, X \in w_X$, and $A \cap B \in w_X$ for all $A, B \in w_X$. Then (X, w_X) is called a w-space. Further, the generalizations of w-open and W-continuity were introduced in [17]. The purpose of this study is to introduce the notion of weak preopen set in diweak texture spaces.

Texture spaces were introduced by L. M. Brown as a point-based setting for the study of fuzzy sets [2]. Ditopologies [1] on textures unify the fuzzy topologies and topologies and in a non-complemented setting by means of duality in the textural concepts, and work continues some applications on mathematical structures such as rough set and soft sets [5–9, 11, 12, 14]. Recently, the notion of diweak texture space based on weak-distructure on texture space was defined in [10]. The aim of this paper is devoted to introduce the generalizations of w-open sets and weak bicontinuity in diweak texture spaces. We investigate some of their properties and the relation between these structures are studied.

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2. TEXTURE SPACES

In this section, we give briefly some basic definitions and results for texture spaces. Full details, motivation and background material may be obtained from to [1–4].

Definition 1. Let U be a set. A *texturing* $\mathcal{U}$ of U is a subset of $\mathcal{P}(U)$ which is a point-separating, complete, completely distributive lattice containing U and $\emptyset$, and for which meet coincides with intersection and finite joins with union. The pair $(U, \mathcal{U})$ is then called a *texture space*, or shortly *texture*.

For $u \in U$, the *p-sets* and the *q-sets* are defined by

$$P_u = \bigcap\{A \in \mathcal{U} \mid u \in A\}, \quad Q_u = \bigvee\{A \in \mathcal{U} \mid u \notin A\}, \quad \text{respectively.}$$

In general a texturing of U need not be closed under set complementation, but it may be that there exists a mapping $\sigma : \mathcal{U} \to \mathcal{U}$ satisfying $\sigma(\sigma(A)) = A,\ \forall A \in \mathcal{U}$ and $A \subseteq B \implies \sigma(B) \subseteq \sigma(A), \forall A, B \in \mathcal{U}$. In this case σ is called a *complementation* on $(U, \mathcal{U})$, and $(U, \mathcal{U}, \sigma)$ is said to be a *complemented texture*.

Example 1. (1) For any set U, $(U, \mathcal{P}(U), c_U)$ is the complemented *discrete texture* representing the usual set structure of U. Here the complementation $c_U(A) = U \setminus A$, $A \subseteq U$, is the usual set complement. Clearly, $P_u = \{u\}$ and $Q_u = U \setminus \{u\}$ for all $u \in U$.

(2) Let $L = (0, 1]$, $\mathcal{L} = \{(0, r] \mid r \in [0, 1]\}$ and $\lambda((0, r]) = (0, 1-r]$, $r \in [0, 1]$. Then $(L, \mathcal{L}, \lambda)$ is complemented texture space. Here $P_r = Q_r = (0, r]$ for all $r \in L$. $(L, \mathcal{L})$ is said to be *Hutton texture*.

(3) For $\mathbb{I} = [0, 1]$ define $\mathcal{J} = \{[0, t] \mid t \in [0, 1]\} \cup \{[0, t) \mid t \in [0, 1]\}$, $\iota([0, t]) = [0, 1-t)$ and $\iota([0, t)) = [0, 1-t]$, $t \in [0, 1]$. Again $(\mathbb{I}, \mathcal{J}, \iota)$ is a complemented texture, which is called *unit interval texture*. Here $P_t = [0, t]$ and $Q_t = [0, t)$ for all $t \in I$.

Difunctions arise often in the study of texture theory. A difunction is a direlation [4] (f, F) satisfying certain additional conditions.

Difunctions: Let (f, F) be a direlation from $(U, \mathcal{U})$ to $(V, \mathcal{V})$. Then (f, F) is called a *difunction from* $(U, \mathcal{U})$ *to* $(V, \mathcal{V})$ if it satisfies the following two conditions.

DF1 For $u, u' \in U$, $P_u \nsubseteq Q_{u'} \implies \exists\, v \in V$ with $f \nsubseteq \overline{Q}_{(u,v)}$ and $\overline{P}_{(u',v)} \nsubseteq F$.

DF2 For $v, v' \in T$ and $u \in U$, $f \nsubseteq \overline{Q}_{(u,v)}$ and $\overline{P}_{(u,v')} \nsubseteq F \implies P_{v'} \nsubseteq Q_v$.

Image and Inverse Image: Let $(f, F) : (U, \mathcal{U}) \to (V, \mathcal{V})$ be a difunction.

(1) For $A \in \mathcal{U}$, the *image* $f^{\to}A$ and the *co-image* $F^{\to}A$ are defined by

$$f^{\to}A = \bigcap\{Q_v \mid \forall u,\ f \not\subseteq \overline{Q}_{(u,v)} \implies A \subseteq Q_u\},$$

$$F^{\to}A = \bigvee\{P_v \mid \forall u,\ \overline{P}_{(u,v)} \not\subseteq F \implies P_u \subseteq A\}.$$

(2) For $B \in \mathcal{V}$, the *inverse image* $f^{\leftarrow}B$ and the *inverse co-image* $F^{\leftarrow}B$ are defined by

$$f^{\leftarrow}B = \bigvee\{P_u \mid \forall v,\ f \not\subseteq \overline{Q}_{(u,v)} \implies P_v \subseteq B\},$$

$$F^{\leftarrow}B = \bigcap\{Q_u \mid \forall v,\ \overline{P}_{(u,v)} \not\subseteq F \implies B \subseteq Q_v\}.$$

For a difunction, the inverse image and the inverse co-image are equal, but the image and co-image are usually not.

Weak-distructure on texture space ([10]): A *weak-distructure* on a texture $(U, \mathcal{U})$ is a pair $(\mathcal{W}, c\mathcal{W})$ of subsets of $\mathcal{U}$, where the set of *weak-open*, or shortly w-open, sets $\mathcal{W}$ satisfies

(1) $U, \emptyset \in \mathcal{W}$,
(2) $G_1, G_2 \in \mathcal{W} \implies G_1 \cap G_2 \in \mathcal{W}$.

and the set of *weak-closed*, or shortly w-closed, sets $c\mathcal{W}$ satisfies

(1) $U, \emptyset \in c\mathcal{W}$,
(2) $K_1, K_2 \in c\mathcal{W} \implies K_1 \cup K_2 \in c\mathcal{W}$.

If $(\mathcal{W}, c\mathcal{W})$ is a weak-distructure on $(U, \mathcal{U})$ then $(U, \mathcal{U}, \mathcal{W}, c\mathcal{W})$ is called *diweak texture space* or shortly, *diw*-texture space.

We denote by $\mathcal{W}O(U)$, the set of w-open sets in $\mathcal{U}$. Likewise, $\mathcal{W}C(U)$ will denote the set of w-closed sets.

For $A \in \mathcal{U}$, the weak-closure and the weak-interior are defined by follows:

$$cl_w(A) = \bigcap\{K \in c\mathcal{W} \mid A \subseteq K\}, \quad \text{and} \quad int_w(A) = \bigvee\{G \in \mathcal{W} \mid G \subseteq A\}.$$

If $(\mathcal{W}, c\mathcal{W})$ is a weak-distructure on a complemented texture $(U, \mathcal{U}, \sigma)$ we say $(\mathcal{W}, c\mathcal{W})$ is complemented if $c\mathcal{W} = \sigma(\mathcal{W})$. In this case we have $\sigma(cl_w(A)) = int_w(\sigma(A))$ and $\sigma(int_w(A)) = cl_w(\sigma(A))$.

w-Bicontinuous difunction: Let $(U_j, \mathcal{U}_j, \mathcal{W}_j, c\mathcal{W}_j)$, $j = 1, 2$, be a diw-texture spaces. Then the difunction $(f, F) : (U_1, \mathcal{U}_1) \to (U_2, \mathcal{U}_2)$ is called

(1) w-continuous if $F^{\leftarrow}(G) \in \mathcal{W}O(U_1)$ for every $G \in \mathcal{W}O(U_2)$.
(2) w-cocontinuous if $f^{\leftarrow}(K) \in \mathcal{W}C(U_1)$ for every $K \in \mathcal{W}C(U_2)$.
(3) w-bicontinuous if it is w-continuous and w-cocontinuous.

3. Preopen sets in diweak texture spaces

Firstly, recall [17] that a subset A of a weak space (X, w_X) is called w-preopen if $A \subseteq wI\, wC(A)$ where $wC\,(wI\,)$ is weak-closure (weak-interior). In the case of a diweak texture space we may give a corresponding definition for w-preopen and w-preclosed sets, as follows:

Definition 2. Let $(U, \mathcal{U}, \mathcal{W}, c\mathcal{W})$ be a diw-texture space. A set $A \in \mathcal{U}$ is called

(1) weak preopen, or shortly w-preopen, if $A \subseteq int_w\, cl_w\,(A)$, and
(2) weak preclosed, or shortly w-preclosed, if $cl_w\, int_w\,(A) \subseteq A$.

We denote by $\mathcal{W}PO(U, \mathcal{U}, \mathcal{W}, c\mathcal{W})$, or when there can be no confusion by $\mathcal{W}PO(U)$, the set of w-preopen sets in $\mathcal{U}$. Likewise, $\mathcal{W}PC(U, \mathcal{U}, \mathcal{W}, c\mathcal{W})$, or $\mathcal{W}PC(U)$ will denote the set of w-preclosed sets.

Lemma 1. *For a given diw-texture space* $(U, \mathcal{U}, \mathcal{W}, c\mathcal{W})$*:*

(i) $\mathcal{W}O(U) \subseteq \mathcal{W}PO(U)$ *and* $\mathcal{W}C(U) \subseteq \mathcal{W}PC(U)$.
(ii) *Arbitrary join of w-preopen sets is w-preopen.*
(iii) *Arbitrary intersection of w-preclosed sets is w-preclosed.*

Proof. (i) Let $G \in \mathcal{W}O(U)$. Since $G \subseteq cl_w(G)$ and $G = int_w(G)$, we have $G = int_w(G) \subseteq int_w(cl_w(G))$. That is, $G \in \mathcal{W}PO(U)$. Likewise, it is proved that $\mathcal{W}C(U) \subseteq \mathcal{W}PC(U)$.

(ii) Let $\{G_j\}$, $j \in J$, be a family of w-preopen sets. Then $G_j \subseteq int_w(cl_w(G_j)) \subseteq int_w(cl_w(\bigvee G_j))$. Thus, $\bigvee G_j \subseteq int_w(cl_w(\bigvee G_j))$, and so $\bigvee G_j$ is w-preopen.

(iii) Let $\{K_j\}$, $j \in J$ be a family of w-preclosed sets. Then $cl_w(int_w(\bigcap K_j)) \subseteq cl_w(int_w(K_j)) \subseteq K_j$. Then $cl_w(int_w(\bigcap K_j)) \subseteq \bigcap K_j$, and so $\bigcap K_j$ is w-preclosed. □

Recall that [10] a subset $A \in \mathcal{U}$ of a diw-texture space $(U, \mathcal{U}, \mathcal{W}, c\mathcal{W})$ is called weak semi-open (weak semi-closed) if $A \subseteq cl_w(\,int_w(A))\,(\,int_w(cl_w(A)) \subseteq A\,)$.

Example 2. Let $U = \{a, b, c, d, e\}$.

(1) In general, the intersection of two w-preopen sets is not w-preopen. Indeed, let us consider the weak-distructure $(\mathcal{W}, c\mathcal{W})$ on the discrete texture $(U, \mathcal{P}(U))$ where $\mathcal{W} = \{U, \{a, b\}, \{c, d\}, \emptyset\}$ and $c\mathcal{W} = \{\emptyset, \{c, d, e\}, \{a, b, e\}, U\}$. For $A = \{a, c, d, e\}$ and $B = \{b, c, d, e\}$, we have $A, B \in \mathcal{W}PO(U)$ and $cl_w(A) = cl_w(B) = U$ and $int_w(cl_w(A \cap B)) = int_w(cl_w(\{c, d, e\})) = \{c, d\}$. But $A \cap B$ is not w-preopen set.

    In general, the union of two w-preclosed sets need not be w-preclosed.

(2) In diw-texture spaces, a w-preopen set can not be weak semi-open. Consider that the diw-structure $(\mathcal{W}, c\mathcal{W})$ on the discrete texture $(U, \mathcal{P}(U))$ where $\mathcal{W} = \{U, \{a, b\}, \{c, d\}, \{b\}, \emptyset\}$ and $c\mathcal{W} = \{\emptyset, \{c, d, e\}, \{a, b, e\}, \{a, c, d, e\}, U\}$. Let $A = \{a, b, d\}$. Clearly, A is not w-open. Since $cl_w(\{a, b, d\}) = U$, B is w-preopen. Further B is not weak semi-open, since $cl_w(int_w(\{a, b, d\}) = cl_w(\{a, b\}) = \{a, b, e\}$.

In general, there is no relation between the w-preopen and w-preclosed sets, but for a complemented diw-texture space we have:

Proposition 1. *For a complemented diw-texture space* $(U, \mathcal{U}, \sigma, \mathcal{W}, c\mathcal{W})$*:*

$$A \in \mathcal{U} \quad \textit{is weak preopen if and only if} \quad \sigma(A) \ \textit{is weak preclosed.}$$

Proof. For $A \in \mathcal{U}$, since $\sigma(int_w(A)) = cl_w(\sigma(A))$ and $\sigma(cl_w(A)) = int_w(\sigma(A))$, the proof is trivial. □

Example 3. (1) If (X, w) is a weak space then $(X, \mathcal{P}(X), c_X, w, w^c)$ is a complemented diw-texture space where $w^c = \{X \setminus G \mid G \in w\}$. Clearly, the weak preopen and weak preclosed sets in (X, w) correspond precisely to the w-preopen and w-preclosed respectively, in $(X, \mathcal{P}(X), c_X, w, w^c)$.

(2) Consider the texture $(L, \mathcal{L})$ of Examples 1 (2). Define the diw-texture $(\mathcal{W}, c\mathcal{W})$ on $(L, \mathcal{L})$ where $\mathcal{W} = \{\emptyset, L\}$ and $c\mathcal{W} = \mathcal{L}$. The only w-preopen sets are $\emptyset$ and L. Dually, let $\mathcal{W} = \mathcal{L}$ and $c\mathcal{W} = \{\emptyset, L\}$. Then the only w-preclosed sets are $\emptyset$ and L.

(3) Consider unit interval texture $(\mathbb{I}, \mathcal{I}, \iota)$. Then $\mathcal{W}_{\mathbb{I}} = \{[0, r) \mid 0 \leq r \leq 1\} \cup \{\mathbb{I}\}$, $c\mathcal{W}_{\mathbb{I}} = \{[0, r] \mid 0 \leq r \leq 1\} \cup \{\emptyset\}$ defines a complemented weak diw-structure on $(\mathbb{I}, \mathcal{I}, \iota)$. Then we have $\mathcal{WPO}(\mathbb{I}) = \mathcal{WPC}(\mathbb{I}) = \mathcal{I}$.

(4) Let (τ, κ) be a ditopology on the texture $(U, \mathcal{U})$. Then, automatically, the pair (τ, κ) is a weak-distructure on $(U, \mathcal{U})$. Hence, preopen and preclosed set in the ditopology (τ, κ) [13] are w-preopen and w-preclosed sets in the diweak texture space $(U, \mathcal{U}, \tau, \kappa)$.

Definition 3. Let $(U, \mathcal{U}, \mathcal{W}, c\mathcal{W})$ be a diw-texture space and $A \in \mathcal{U}$. We define:

(1) The weak pre-closure $pcl_w(A)$ of A under $(\mathcal{W}, c\mathcal{W})$ by the equality

$$pcl_w(A) = \bigcap \{B \mid B \in \mathcal{WPC}(U) \text{ and } A \subseteq B\}.$$

(2) The weak pre-interior $pint_w(A)$ of A under $(\mathcal{W}, c\mathcal{W})$ by the equality

$$pint_w(A) = \bigvee \{B \mid B \in \mathcal{WPO}(U) \text{ and } B \subseteq A\}.$$

Note that, by Lemma 1, we have $pcl_w(A) \in \mathcal{WPC}(U)$ and $pint_w(A) \in \mathcal{WPO}(U)$, while $A \in \mathcal{WPC}(U) \iff A = pcl_w(A)$ and $A \in \mathcal{WPO}(U) \iff A = pint_w(A)$.

Obviously, $pcl_w(A)$ is the smallest w-preclosed set which contains A and $pint_w(A)$ is the greatest w-preopen set which is contained in A, and we have $A \subseteq pcl_w(A) \subseteq cl_w(A)$ and $int_w(A) \subseteq pint_w(A) \subseteq A$, by Lemma 1 (i).

Now, we recall that a function between weak spaces is called weak precontinuous [17] if the inverse image of each weak open set is weak preopen. This leads to the following concepts for a difunction between diweak texture spaces.

Definition 4. Let $(U_j, \mathcal{U}_j, \mathcal{W}_j, c\mathcal{W}_j)$, $j = 1, 2$, be a diw-texture spaces. Then the difunction $(f, F) : (U_1, \mathcal{U}_1) \to (U_2, \mathcal{U}_2)$ is called

(1) w-precontinuous if $F^{\leftarrow}(G) \in \mathcal{W}PO(U_1)$ for every $G \in \mathcal{W}O(U_2)$.
(2) w-precocontinuous if $f^{\leftarrow}(K) \in \mathcal{W}PC(U_1)$ for every $K \in \mathcal{W}C(U_2)$.
(3) w-prebicontinuous if it is w-precontinuous and w-precocontinuous.

Since every w-open (w-closed) set is w-preopen (w-preclosed) set, every w-bicontinuous difunction is w-prebicontinuous.

Remark 1. Recall that [4] if $f : X \to Y$ is a point function then (f, f') is a difunction from $(X, \mathcal{P}(X))$ to $(Y, \mathcal{P}(Y))$ where $f' = (X \times Y) \setminus f$. Conversely, if (f, F) is a difunction from $(X, \mathcal{P}(X))$ to $(Y, \mathcal{P}(Y))$ then $F = (X \times Y) \setminus f$ and $F^{\leftarrow} = f^{-1}$.

Consequently, f is weak precontinuous point function from (X, w_X) to (Y, w_Y) if and only if (f, f') is w-prebicontinuous difunction from $(X, \mathcal{P}(X), w_X, {w_X}^c)$ to $(Y, \mathcal{P}(Y), w_Y, {w_Y}^c)$.

Note that a function between topological spaces is called M-precontinuous [15] if the inverse image of each preopen set is preopen. In the case of a diweak texture space we may give a corresponding definition for M-bicontinuity, as follows:

Definition 5. Let $(U_j, \mathcal{U}_j, \mathcal{W}_j, c\mathcal{W}_j)$, $j = 1, 2$, be a diw-texture spaces. Then the difunction $(f, F) : (U_1, \mathcal{U}_1) \to (U_2, \mathcal{U}_2)$ is called

(1) wM-precontinuous if $F^{\leftarrow}(G) \in \mathcal{W}PO(U_1)$ for every $G \in \mathcal{W}PO(U_2)$.
(2) wM-precocontinuous if $f^{\leftarrow}(K) \in \mathcal{W}PC(U_1)$ for every $K \in \mathcal{W}PC(U_2)$.
(3) wM-prebicontinuous if it is weak wM-precontinuous and weak wM-precocontinuous.

Likewise, we can define the concept of strongly bicontinuity in diweak texture spaces as follows:

Definition 6. Let $(U_j, \mathcal{U}_j, \mathcal{W}_j, c\mathcal{W}_j)$, $j = 1, 2$, be a diw-texture spaces. Then the difunction $(f, F) : (U_1, \mathcal{U}_1) \to (U_2, \mathcal{U}_2)$ is called

(1) strongly w-precontinuous if $F^{\leftarrow}(G) \in \mathcal{W}O(U_1)$ for every $G \in \mathcal{W}PO(U_2)$.
(2) strongly w-precocontinuous if $f^{\leftarrow}(K) \in \mathcal{W}C(U_1)$ for every $K \in \mathcal{W}PC(U_2)$.
(3) strongly w-prebicontinuous if it is strongly w-precontinuous and strongly w-precocontinuous.

Proposition 2. *Let $(U_j, \mathcal{U}_j, \mathcal{W}_j, c\mathcal{W}_j)$, $j = 1, 2$, be a diw-texture spaces and $(f, F) : (U_1, \mathcal{U}_1) \to (U_2, \mathcal{U}_2)$ be a difunction. Then (f, F) is*

(1) *strongly w-prebicontinuous $\Longrightarrow$ w-bicontinuous $\Longrightarrow$ w-prebicontinuous.*
(2) *strongly w-prebicontinuous $\Longrightarrow$ wM-prebicontinuous $\Longrightarrow$ w-prebicontinuous.*

Proof. (1) Let (f, F) be a strongly w-precontinuous difunction. We take $G \in \mathcal{W}O(U_2)$. Then G is w-preopen, and we have $F^{\leftarrow}(G) \in \mathcal{W}O(U_1)$ since (f, F) is strongly w-precontinuous.

Secondly, let (f, F) be a w-continuous and $G \in \mathcal{W}O(U_2)$. Since (f, F) is a w-continuous, we have $F^{\leftarrow}(G) \in \mathcal{W}O(U_1)$ and so $F^{\leftarrow}(G)$ is a w-preopen set.

The proof of the w-cocontinuity is dual and omitted.

(2) Firstly, we suppose that (f, F) is strongly w-prebicontinuous. Let $G \in \mathcal{W}PO(U_2)$. Since (f, F) is strongly w-precontinuous, $F^{\leftarrow}(G) \in \mathcal{W}O(U_1)$, and so $F^{\leftarrow}(G)$ is w-preopen.

Secondly, suppose that (f, F) is a wM-precontinuous difunction. Let $G \in \mathcal{W}O(U_2)$. Then $F^{\leftarrow}(G)$ is w-preopen, since G is w-preopen and (f, F) is wM-precontinuous.

The proof of the results

$$\begin{aligned} \text{strongly } w\text{-precocontinuous} \quad &\Longrightarrow \quad wM - \text{precocontinuous} \\ &\Longrightarrow \quad w\text{-precocontinuous} \end{aligned}$$

is dual of the above and omitted. □

The following proposition gives a characterization for wM-precontinuity and M-precocontinuity.

Proposition 3. *Let $(U_j, \mathcal{U}_j, \mathcal{W}_j, c\mathcal{W}_j)$, $j = 1, 2$, be a diw-texture spaces and $(f, F) : (U_1, \mathcal{U}_1) \to (U_2, \mathcal{U}_2)$ be a difunction.*

(1) *The following are equivalent:*
 (i) *(f, F) is wM-precontinuous.*
 (ii) *$A \in \mathcal{U}_1 \Longrightarrow pint_w(F^{\to}A) \subseteq F^{\to}(pint_w(A))$.*
 (iii) *$B \in \mathcal{U}_2 \Longrightarrow F^{\leftarrow}(pint_w(B)) \subseteq pint_w(F^{\leftarrow}B)$.*

(2) *The following are equivalent:*
 (i) *(f, F) is wM-precocontinuous.*
 (ii) *$A \in \mathcal{U}_1 \Longrightarrow f^{\to}(pcl_w(A)) \subseteq pcl_w(f^{\to}(A))$.*
 (iii) *$B \in \mathcal{U}_2 \Longrightarrow pcl_w(f^{\leftarrow}(B)) \subseteq f^{\leftarrow}(pcl_w(B))$.*

Proof. We prove (2), leaving the dual proof of (1) to the interested reader.

(i)$\Longrightarrow$ (ii) Let (f, F) be a wM-precocontinuous difunction. Take $A \in \mathcal{U}_1$. Since $pcl_w(f^{\to}(A)) \in \mathcal{W}PC(U_2)$, $f^{\leftarrow}(pcl_w(f^{\to}(A))) \in \mathcal{W}PC(U_1)$. By [4, Lemma 2.9], we have $A \subseteq f^{\leftarrow}(f^{\to}A) \subseteq f^{\leftarrow}(pcl_w(f^{\to}(A)))$. Since $pcl_w(A) \subseteq f^{\leftarrow}(pcl_w(f^{\to}(A)))$, we have

$$f^{\to}(pcl_w(A)) \subseteq f^{\to}(f^{\leftarrow}(pcl_w(f^{\to}(A)))) \subseteq pcl_w(f^{\to}(A)).$$

(ii)$\Longrightarrow$ (iii) Let $B \in \mathcal{U}_2$. Then $A := f^{\leftarrow}(B) \in \mathcal{U}_1$. From (ii), we have $f^{\to}(pcl_w(A)) \subseteq pcl_w(f^{\to}(A))$. Then

$$f^{\to}(pcl_w(f^{\leftarrow}(B))) \subseteq pcl_w(f^{\to}(f^{\leftarrow}(B)) \subseteq pcl_w(B)$$

by [4, Lemma 2.9]. Hence,

$$pcl_w(f^{\leftarrow}(B)) \subseteq f^{\leftarrow}(f^{\rightarrow}(pcl_w(f^{\leftarrow}(B)))) \subseteq f^{\leftarrow}(pcl_w(B))$$

which proves (iii).

(iii)$\Longrightarrow$ (i) We prove that (f, F) is wM-precocontinuous. Let $B \in \mathcal{W}PC(U_2)$. Since B is w-preclosed, we have

$$pcl_w(f^{\leftarrow}(B)) \subseteq f^{\leftarrow}(pcl_w(B)) \subseteq f^{\leftarrow}(B)$$

from assumption (iii). Consequently, $f^{\leftarrow}(B)$ is w-preclosed, and (f, F) is wM-precocontinuous. □

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References

[1] Brown, L. M. and Diker, M. *Ditopological texture spaces and intuitionistic sets*, Fuzzy sets and systems **98**, (1998), 217–224.

[2] Brown, L. M. and Ertürk, R. *Fuzzy Sets as Texture Spaces, I. Representation Theorems*, Fuzzy Sets and Systems **110** (2) (2000), 227–236.

[3] Brown, L. M. and Ertürk, R. *Fuzzy sets as texture spaces, II. Subtextures and quotient textures*, Fuzzy Sets and Systems **110** (2) (2000), 237–245.

[4] Brown, L. M., Ertürk, R. and Dost, Ş. *Ditopological texture spaces and fuzzy topology, I. Basic Concepts*, Fuzzy Sets and Systems **147** (2) (2004), 171–199.

[5] Dost, Ş. *Textures and Base spaces*, South Asian Journal of Mathematics, **9** (1) (2019), 12–18.

[6] Dost, Ş. *Precise Sets in Approximation Spaces and Textures*, Fundamenta Informaticae, **158** (4) (2018), 353–368.

[7] Dost, Ş. *Metric spaces and textures*, Applied General Topology, **18** (1) (2017), 203–217.

[8] Dost, Ş. *A textural view of soft fuzzy rough sets*, Journal of Intelligent & Fuzzy Systems, **28** (6) (2016), 2519–2535.

[9] Dost, Ş. *Serial relation and textural rough set*, Mathematica Moravica, **25** (2) (2021), 69–79.

[10] Dost, Ş. *Weak structures on texture spaces and weak semiopen sets*, Boletim da Sociedade Paranaense de Matematica, In press.

[11] Ekmekçi, R. *Graded Diextremities*, Filomat, 32 (1) (2018), 149–164.

[12] Ekmekçi, R., Ertürk, R. *Q-Convergence in graded ditopological texture spaces*, Mathematica Moravica, 21 (1) (2017), 27–36.

[13] Gohar, M.M., Brown, L.M. *Strong compactness of ditopological texture spaces*, Int. Conf. Math. Sci., American Institute of Physics, (2010), 153–168.

[14] Kule, M., Dost, Ş. *A textural view of soft fuzzy rough sets*, Annals of Fuzzy Mathematics and Informatics, **12** (1) (2016), 47–62.

[15] Mashhour, A. S., Abd-El-Monsef, M.E. and Hasanein, I. A. *On pretopological spaces*, Bull. Math. Soc. , 138 (1969), 159–170.

[16] Min, W. K. and Kim, Y. K. *On Weak Structures and w- spaces*, Far East Journal of Mathematical Sciences, 97(5) (2015), 549-561.

[17] Min, W. K. and Kim, Y. K. *w-preopen sets and W-precontinuity in weak spaces*, Int. Journal of Mathematical Analysis, **10** (21) (2016), 1009–1017.

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